Nonlinear Schrödinger Equation in Foundations: Summary of 4 Catches

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Abstract

Fundamental modifications of the standard Schrödinger equation by additional nonlinear terms have been considered for various purposes over the recent decades. It came as a surprise when, inverting Abner Shimonyi's observation of "peaceful coexistence" between standard quantum mechanics and relativity, N. Gisin proved in 1990 that any (deterministic) nonlinear Schrödinger equation would allow for superluminal communication. This is by now the most spectacular and best known fundational anomaly. I am going to discuss further anomalies, simple but foundational, less spectacular but not less dramatic.

Peaceful coexistence ...

of quantum mechanics and special relativity (Shimony)

Despite

- apparent action-at-a-distance in EPR situation
- quantum non-locality in Bell formulation

action-at-a-distance (AAD) & faster-then-light (FTL) communication remain impossible.

Reason: linear structure of quantum mechanics
Non-linear modifications open door to FTL communication! (Gisin)

$$i\hbar\frac{d\psi}{dt}=\hat{H}\psi+\hat{V}_\psi\psi$$

allows for FTL communication for whatever small (non-trivial) $\hat{V}_\Psi$.

Schrödinger-Newton Equation - our testbed

Single-body SNE for c.o.m. free motion of "large" mass M:

$$i\hbar\frac{d\psi}{dt} = \frac{\hat{p}^2}{2M}\psi + M\Phi_\psi(\hat{x})\psi, \qquad \Phi_\psi(\hat{x}) = -GM\int\frac{|\psi(r)|^2}{|\hat{x}-r|}d^3r$$

May be foundational (D., Penrose)

- Stationary solution: single soliton $\bigcirc$ of $\emptyset \sim (\hbar^2/GM^3)$
- Schrödinger Cat state: two-soliton $\psi_\pm = \bigcirc_L \pm \bigcirc_R$

By mean-field $\Phi_\psi(\hat{x})$, parts in $\psi_\pm$ attract each other, like, e.g.:

$$\bigcirc \qquad \bigcirc \Longrightarrow \bigcirc\bigcirc \Longrightarrow \bigcirc\!\!\!\bigcirc \Longrightarrow \bigcirc\bigcirc \Longrightarrow \bigcirc \qquad \bigcirc$$

1-solitons $\bigcirc_L$ and $\bigcirc_R$ are static, 2-solitons $\psi_\pm = \bigcirc_L \pm \bigcirc_R$ evolve.
Initial overlap is $1/\sqrt{2}$. SNE makes them orthogonal after time

$$\sim \frac{\hbar}{GM^2}d_{L-R}$$

Schrödinger-Newton Fake AAD - catch 1

(after Gisin's 2−qubit FTL telegraph 1990)
0) Alice and Bob are far away from each other.
1) Alice owns qubit, Bob owns M, in entagled state:

$$\uparrow_z \otimes \bigcirc_L + \downarrow_z \otimes \bigcirc_R$$

2) Alice measures either $\hat{\sigma}_z$ or $\hat{\sigma}_x$
If she measures $\hat{\sigma}_z$
$\Longrightarrow$ Bob's state collapses into static single soliton $\bigcirc_L$ or $\bigcirc_R$
If she measures $\hat{\sigma}_x$
$\Longrightarrow$ Bob' state collapses into 2-soliton superposition $\bigcirc_L \pm \bigcirc_R$
which evolves to become orthogonal to both $\bigcirc_L, \bigcirc_R$
3) Using no physical interaction, Alice achieved AAD, detectable with certainty by Bob.

Schrödinger-Newton FTL Telegraph - catch 2

Same as SN Fake AAD, with timing conditions:
After time of orthogonalization

$$\sim \frac{\hbar}{GM^2} d_{L-R}$$

between single and double-soliton states,
Bob can (via $\hat{x}$-measurement) distinguish with certainty between Alice choices σ_z- or σ_x-measurements.
If orthogonalization time is shorter than light time-of-flight from Alice to Bob, then AAD has allowed for FTL.

Schrödinger-Newton Eq. Can't Evolve Mixed State - catch 3

0) Alice and Bob are far away from and don't know about each other.
1) Alice owns qubit, Bob owns M, in entagled state:

$$\uparrow_z \otimes \bigcirc_L + \downarrow_z \otimes \bigcirc_R$$

2) Alice does not measure anything.
$\Longrightarrow$ Bob's local state is a mixed state, descibed by

$$\hat{\rho} = \frac{1}{2}\Big(|\bigcirc_L\rangle\langle\bigcirc_L| + |\bigcirc_L\rangle\langle\bigcirc_R| + |\bigcirc_R\rangle\langle\bigcirc_L| + |\bigcirc_R\rangle\langle\bigcirc_R|\Big)$$

3) SNE does not apply to density matrices but to state vectors.
$\Longrightarrow$ Bob can not calculate the dynamics of his system.

Statistical Interpretation Forbids Non-Linearity - catch 4

- Suppose any **dynamics** $\mathcal{M}$, not necessarily linear or deterministic:
$$\hat{\rho}^f = \mathcal{M}[\hat{\rho}^i]$$
- Consider statistical **mixing** of $\hat{\rho}_1, \hat{\rho}_2$ with weights $\lambda_1 + \lambda_2 = 1$:
$$\hat{\rho} = \lambda_1\hat{\rho}_1 + \lambda_2\hat{\rho}_2$$

In von Neumann standard theory

mixing and dynamics are interchangeable:
$$\mathcal{M}[\lambda_1\hat{\rho}_1 + \lambda_2\hat{\rho}_2] = \lambda_1\mathcal{M}[\hat{\rho}_1] + \lambda_2\mathcal{M}[\hat{\rho}_2]$$
Recognize the condition of $\mathcal{M}$'s linearity!

- Interchangeability excludes non-linear Schrödinger equations
- Without interchangeability statistical interpretation collapses

Catch 4 is non-quantum, it's classical statistical! (D.: *A Short Course in Quantum Information Theory*, Springer, 2007, 2011)

Summary: Catches and Loopholes

Non-linear Schrödinger equations deserve attention in foundations.
Just we should keep in mind catches:
Non-linear Schrödinger equations

- allow for
  - fake action-at-a-distance (maybe extreme weak)
  - faster-than-light communication (maybe too hard to realize)
- does not allow for
  - local dynamics (unless you prepare a pure state)
  - statistical interpretation (maybe a substitute works?)