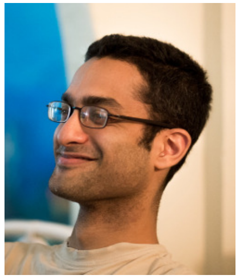
Griffith UNIVERSITY
Centre for Quantum Dynamics

The Two Bell's Theorems of John Bell and Causal Emergence

Eric Cavalcanti
Oct 2015

Howard Wiseman



Raymond Lal

Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km

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- "For more than 80 years, the counterintuitive predictions of quantum theory have stimulated debate about the nature of reality. In his seminal work, John Bell proved that no theory of nature that obeys ***locality*** and ***realism*** can reproduce all the predictions of quantum theory." [my emphasis]

Hensen *et al.'s* concepts

- ``Locality'': "Physical influences do not propagate faster than light''
- ``Realism'': "Physical properties are defined prior to and independent of observation"
- (Plus the usual assumption of "Free Will")

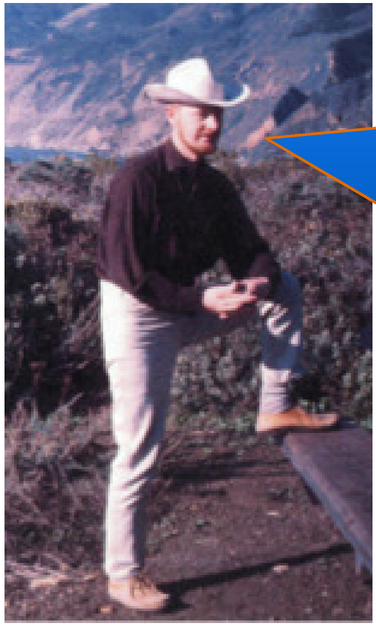
Operationalists vs Realists

Bell's theorem has two assumptions. The first is locality. Quantum Mechanics respects this. The second is something else which I call classicality but which is also called determinism or realism. Clearly it is this that we should abandon. Locality is here to stay.

Bell's theorem uses only one assumption: locality (or, "local causality" as Bell sometimes called it).

Standard quantum mechanics is non-local. Bell's theorem proves that **the world itself is nonlocal**.

What *exactly* did Bell prove in 1964?



In a theory in which **parameters are added** to quantum mechanics to **determine the results** of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the **setting of one measuring device can influence the reading of another instrument**, however remote. (1964)

i.e. quantum correlations violate the joint assumptions of:

- "Predetermination" (of measurement results).
- "Locality, [meaning] that the result of a measurement on one system be unaffected by operations on a distant system".

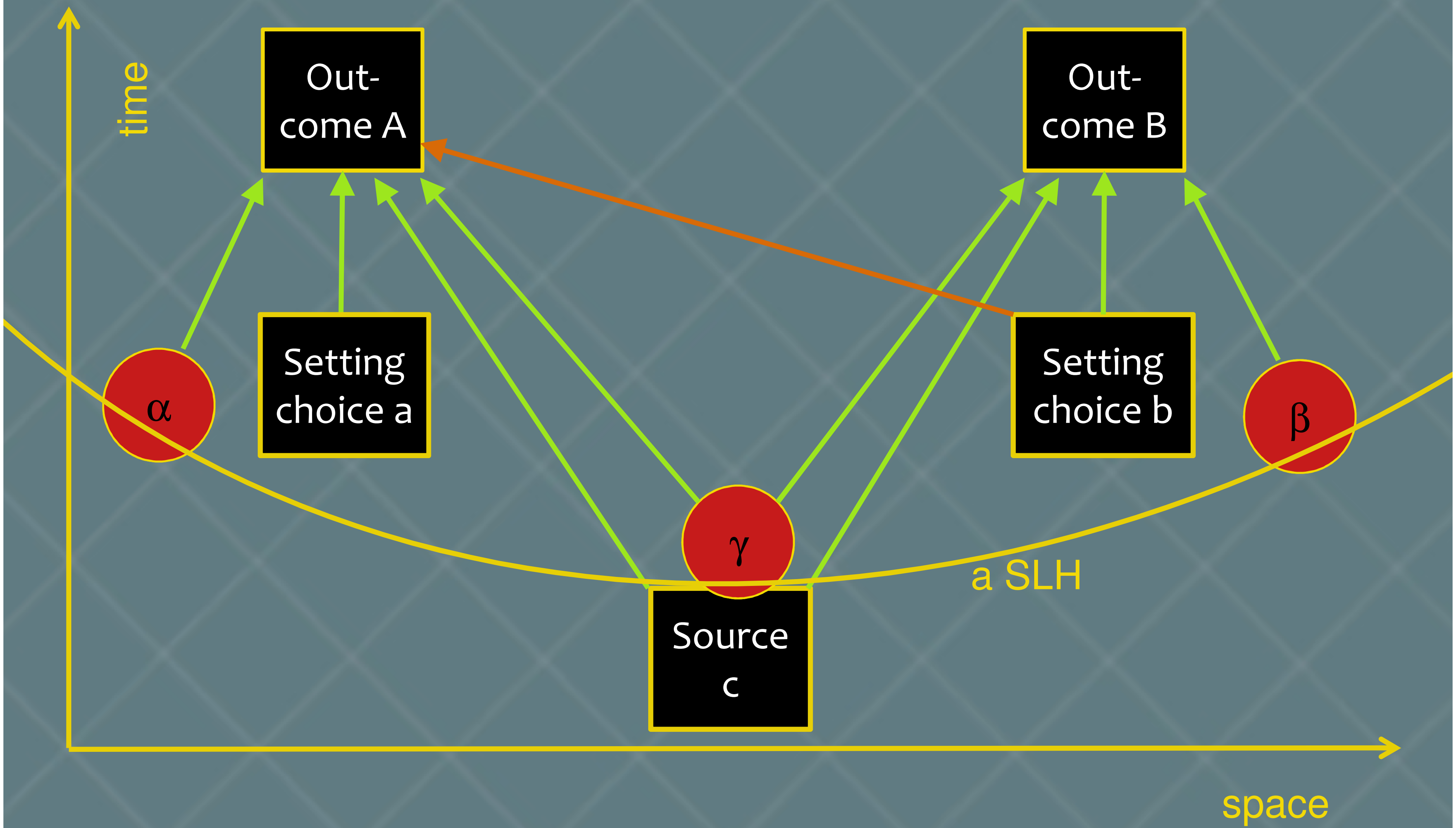
What *exactly* did Bell prove in 1976?

My … notion of **local causality** is that events in [one lab] should not be causes of events in [a *space-like separated* lab], and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the *overlap in their backward light cones*. (1976)

A consequence … of ‘local causality’ [is] the outcomes [in the two labs] having **no [statistical] dependence on one another nor on the settings** of the remote [measurement], but only on the local [measurement settings] and on the [common] past causes. (1990)

Quantum mechanics … gives certain **correlations** which … **cannot be [reproduced by] a locally causal theory**. (1976)

Minkowski Diagram



In terms of probabilities …

- A **quantum phenomenon** is described by a **theory** if

  $P(A,B|a,b,c) = \Sigma_\lambda\, \mu(\lambda|c)\, \xi(A,B|a,b,c,\lambda)$, where $\lambda=(\alpha,\beta,\gamma)$

  [where we assume **no-superdeterminism**: $\mu(\lambda|a,b,c) = \mu(\lambda|c)$]

  - **Predetermination (PD)**: $\xi(A,B|a,b,c,\lambda) = 0$ or 1.
    < **Predictability (P)**: $P(A,B|a,b,c) = 0$ or 1.
  - **Locality (L)**: $\xi(A|a,b,c,\lambda) = P(A|a,c,\lambda)$.
    > **Signal locality (SL)**: $P(A|a,b,c) = P(A|a,c)$.

- **Bell 1964**: there is no theory reproducing quantum phenomena satisfying **locality** and **predetermination**.

  - **Local causality (LC)**: $\xi(A|a,B,b,c,\lambda) = \xi(A|a,\alpha,c,\gamma)$.

- **Bell 1976**: **L** & **PD** can be replaced by **local causality**.

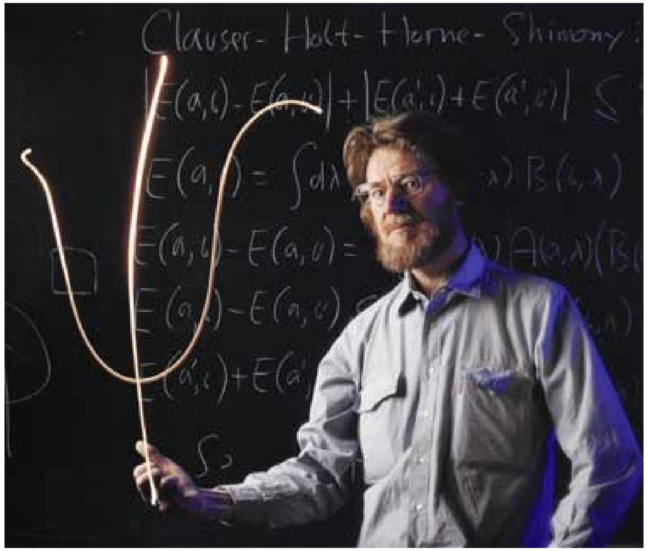
Well that clears that up …

Unfortunately the confusion started even in 1976:

- Having defined “local causality” Bell immediately started using “locality” as a synonym for it, even though it was different from “locality” as he had used in 1964.
- Soon afterwards, Bell started claiming that by “locality” he had *always* meant local causality, and that this was the “sacred principle” which Einstein had believed in.
- Followers of Bell therefore often state that Bell’s theorem says that quantum phenomena are nonlocal.
- Most quantum physicists only know Bell’s 1964 theorem, and say that we can keep locality if we give up predetermination.

"For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation [of quantum theory] and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory . . .

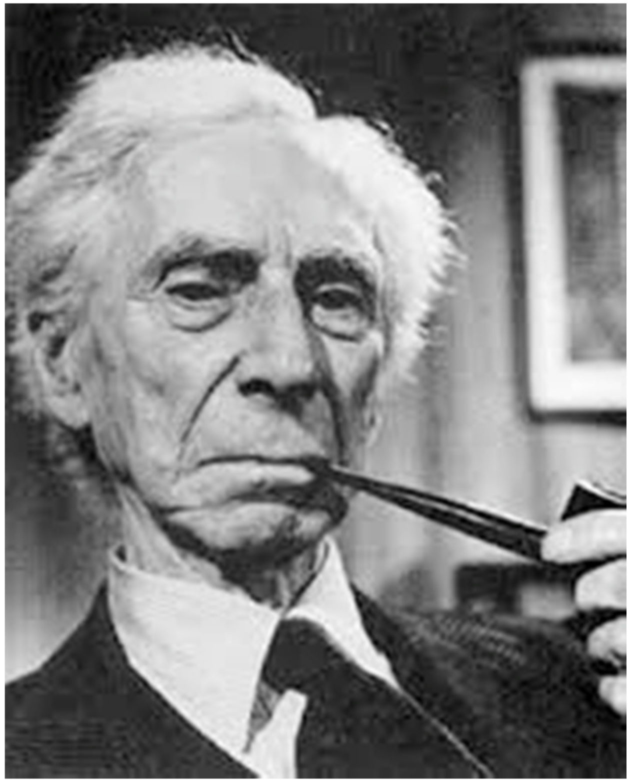
It may be that a real synthesis of quantum and relativity theories requires not just technical developments but radical conceptual renewal."- J.S. Bell, 1986



It’s about causality

- We’ll argue that part of that “radical conceptual renewal” is about our understanding of causation.
- Bell’s theorem is best understood as being about the constraints from causal structure to possible correlations among events.
- Relativity comes in “merely” to supply the causal structure.

Who cares about causality?



"The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm".

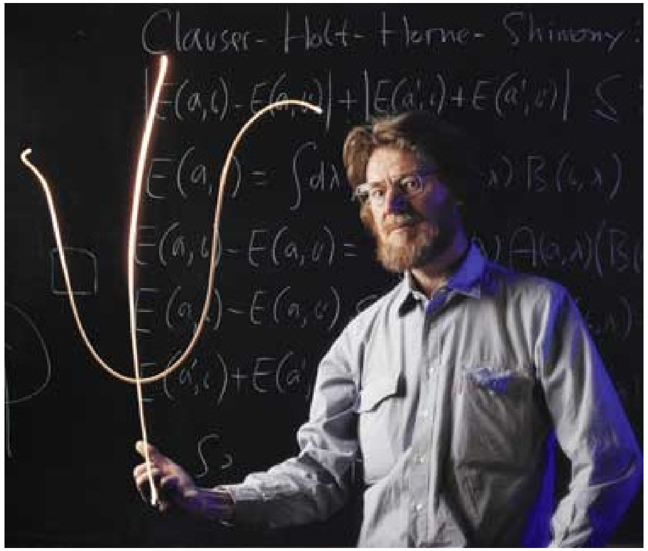
– Bertrand Russel, 'On the Notion of Cause', 1913.

"Bertrand Russell argued that laws of association are all the laws there are, and that causal principles cannot be derived from the causally symmetric laws of association. (…) Causal principles cannot be reduced to laws of association; but they cannot be done away with."

- Nancy Cartwright, 'Causal Laws and Effective Strategies', 1979.

“Do we then have to fall back on “no signalling faster than light" as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have **lost the idea that correlations can be explained** . . .”

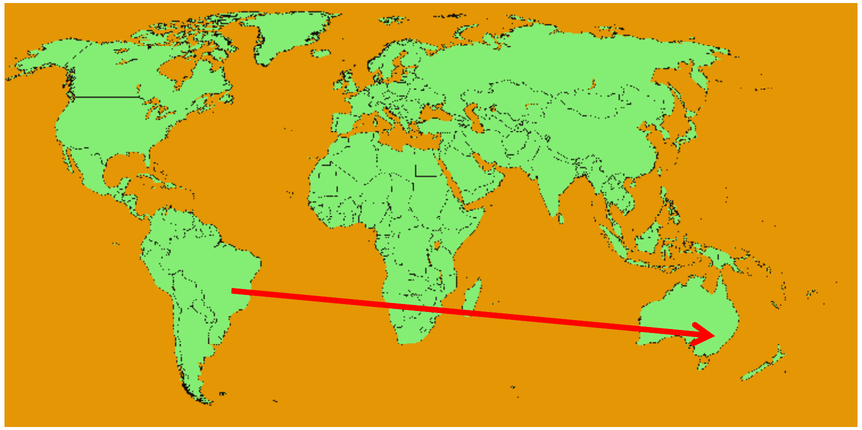
– J.S. Bell

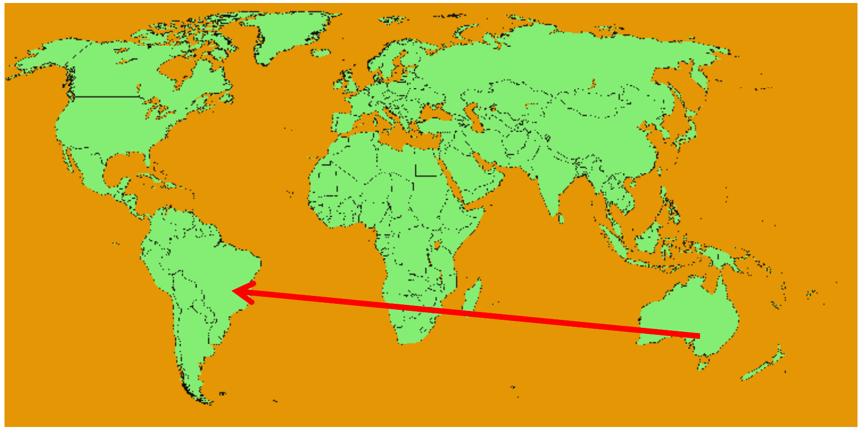


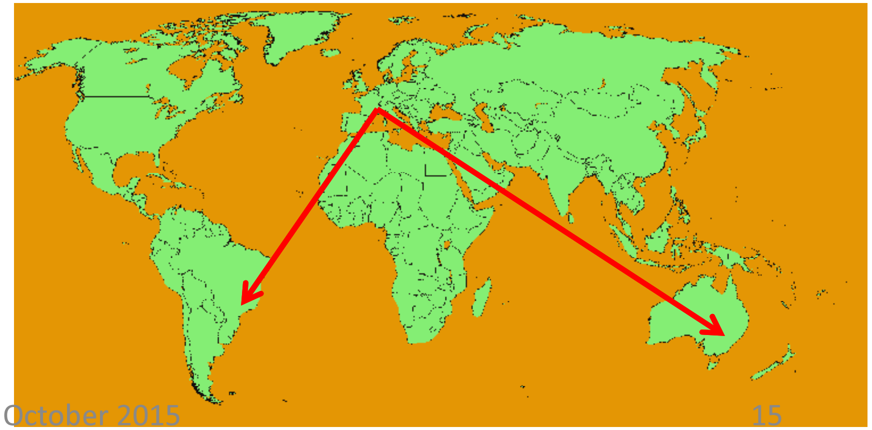
Explaining correlations

Suppose a rare virus infection spreads almost simultaneously in Australia and Brazil.
Possible explanations epidemiologists would entertain:

1) Virus carried from Australia to Brazil;
2) Virus carried from Brazil to Australia;
3) Virus was carried to both places from a third country.







Reichenbach's Principle of Common Cause

Informally: if two events are correlated, either one is a direct cause of the other, or they share a common cause*.

- Fundamental in many areas of science, from medical research to causal discovery algorithms.

- BUT: leads to Bell's theorem and contradiction with quantum theory.

  (Curiously, Reichenbach thought deeply about QM but didn't come to Bell's theorem*)

* H. Reichenbach. Philosophic Foundations of Quantum Mechanics, 1944.

Back to basics …

- **AXIOM 1: MACROREALITY.**
  In an individual run, exactly one outcome A and exactly one outcome B really happen, and are not 'relative' to anything.

- **AXIOM 2: MINKOWSKI SPACE-TIME.**
  Concepts like light-cones, space-like separated (SLS), space-like hypersurfaces (SLHs) *etc.* can be applied unambiguously in ordinary laboratory situation.

- **AXIOM 3: DIRECTED CAUSALITY (CAUSAL ARROW OF TIME).**
  A cause can only be in the past of its effect.

- **POSTULATE: RELATIVISTIC CAUSALITY**
  In **Axiom 3** (**directed causality**), 'the past' is to be understood as 'the past light cone'.

Operationalist Principles

- **PRINCIPLE: AGENT-CAUSATION**
  If an event A is statistically dependent on a freely chosen action b, then b is a cause of A.

- **LEMMA: Relativistic causality** plus **agent-causation** imply **locality**, and **no-superdeterminism**, where
  - **LOCALITY** = The probability of an observable event A is unchanged by conditioning on a SLS free choice b, even if it is already conditioned on other events as long as those events are not in the future light cone of b.
  - **NO-SUPERDETERMINISM** = Any free choices subsequent to a SLH are uncorrelated with events on that SLH.

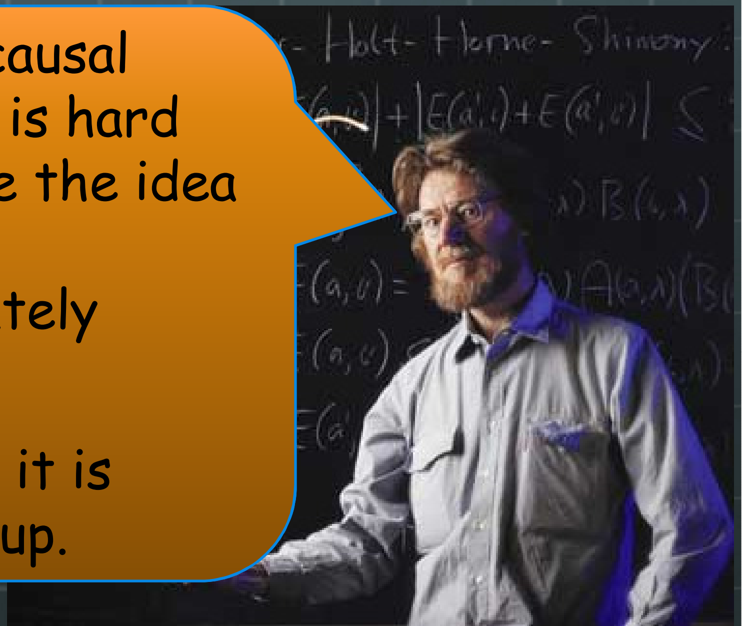
- Remember: **Locality** is respected by orthodox QM.

Operationalist Theorem

- **PRINCIPLE: PREDETERMINATION**. For any observable event A, and any SLH S prior to it, A has causes on S which, together with free choices subsequent to S, determine A.
- **THEOREM ~ Bell (1964):**
  The conjunction of **Relativistic causality, agent-causation**, and **predetermination** is violated by quantum phenomena.

To use **agent causation** as *the* fundamental causal concept in contemporary theoretical physics is hard for me to accept. For one thing we would lose the idea that correlations must be explained. More importantly, it is desperately vague, immediately provoking the question: "What is an agent?"

Also, **predetermination** is a sacrificial lamb – it is unnecessarily strong, and so too easily given up.



Realist Principles

- **REICHENBACH'S PRINCIPLE** (1956):
  If two events *A* and *B* are correlated, then either one is a cause of the other, or they have a common cause C , such that conditioning on *C* eliminates the correlation.

- **LEMMA: Relativistic causality** plus **Reichenbach's Principle** implies **local causality**, where
  - **LOCAL CAUSALITY =** If two SLS events *A* and *B* are correlated, then there is an event *C* in the overlap of their past light cones such that conditioning on *C* eliminates the correlation.

- **PRINCIPLE: FREE CHOICE.**
  A freely chosen action (a or b) has no causes.

Realist Theorem

- **THEOREM ~ Bell (1976)**
  The conjunction of **Relativistic causality, Reichenbach's Principle** and **Free Choice** is contradicted by quantum phenomena.

- For realists, **Relativistic causality** has to be sacrificed to save **Reichenbach** (common causes that explain correlations).

As an operationalist, I find Reichenbach's Principle too strong as the fundamental notion of causation. Why should all correlations in nature be explicable in the way that they are in classical physics?

Also, how can you realists criticize us for having an agent-centric notion of causation when you still need to assume **Free Choice**, which is agent-centric too?

Reconciliation

- **The key is to break***
  - **REICHENBACH'S PRINCIPLE** (1956): If two events *A* and *B* are correlated, then either one is a cause of the other, or they have a common cause *C* , such that conditioning on *C* eliminates the correlation.

into

  - **PRINCIPLE of COMMON CAUSE:** If two events *A* and *B* are correlated, then either one is a cause of the other, or they have a common cause C that explains the correlation.

plus

  - **PRINCIPLE of DECORRELATING EXPLANATION:** A cause C, common to events *A* and *B*, explains a correlation between them only if conditioning on *C* eliminates the correlation.

- *Cavalcanti and Lal, JPA 47, 424018 (2014)

Aside: “Non-commutative common causes”

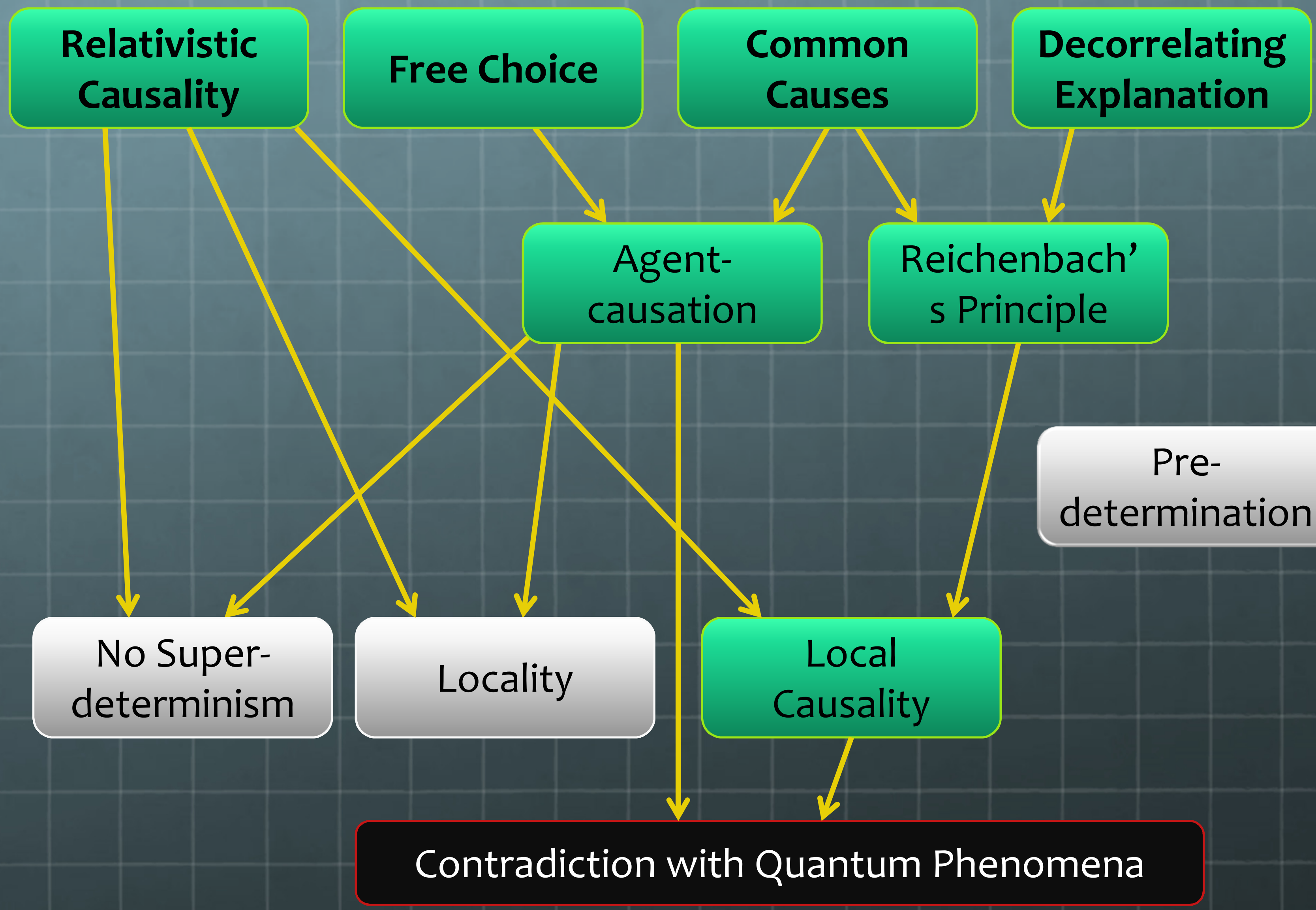
- Hofer-Szabó and Vecsernyés

- We can show* that **any product state** can satisfy the requirements for a “non-commutative common cause” for the correlations of **any quantum state.**

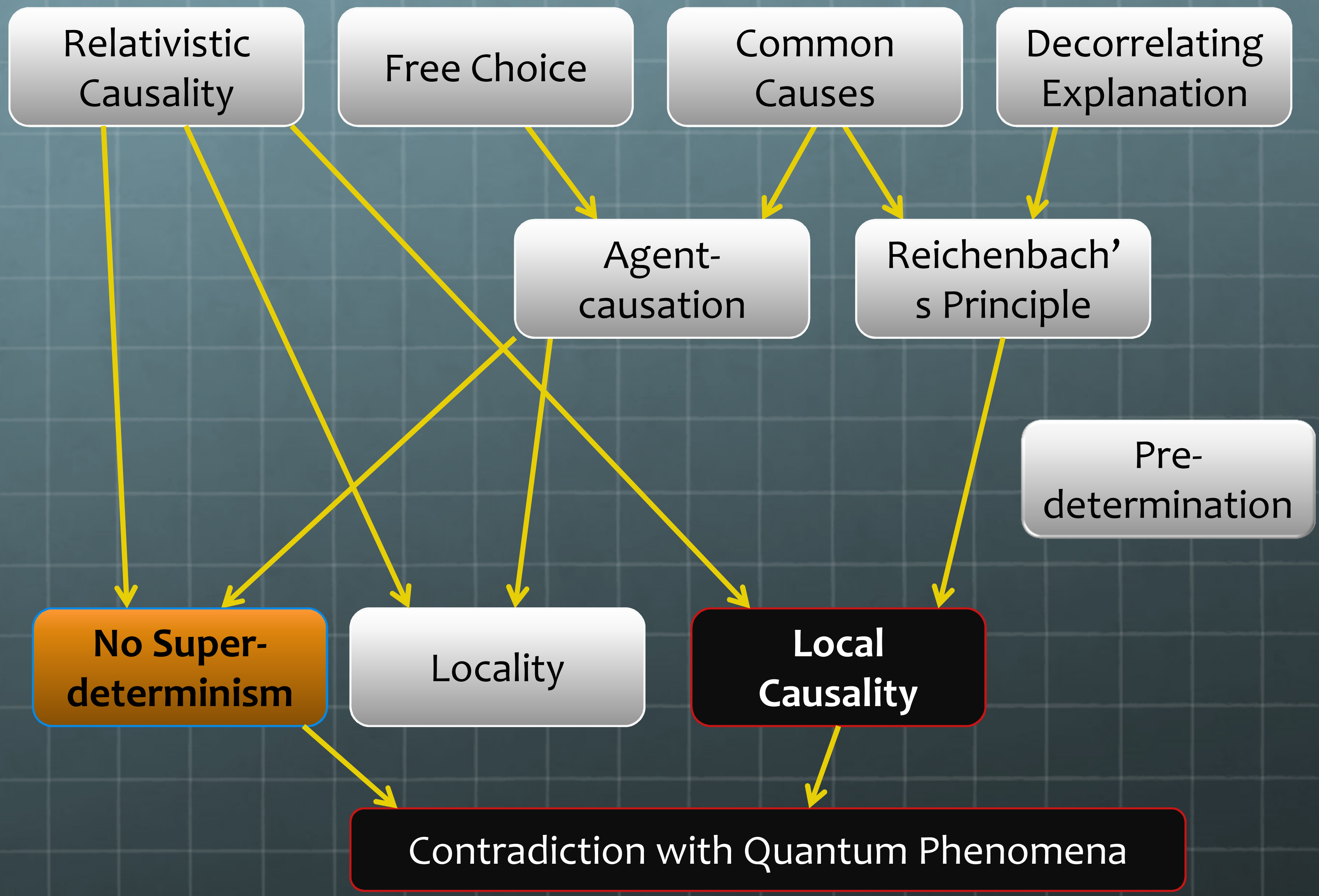
- There is **no deductive or causal link** between the “common causes” and their supposed effects. Non-commutative common causes **do not explain** the correlations.

- *Cavalcanti and Lal, JPA 47, 424018 (2014)

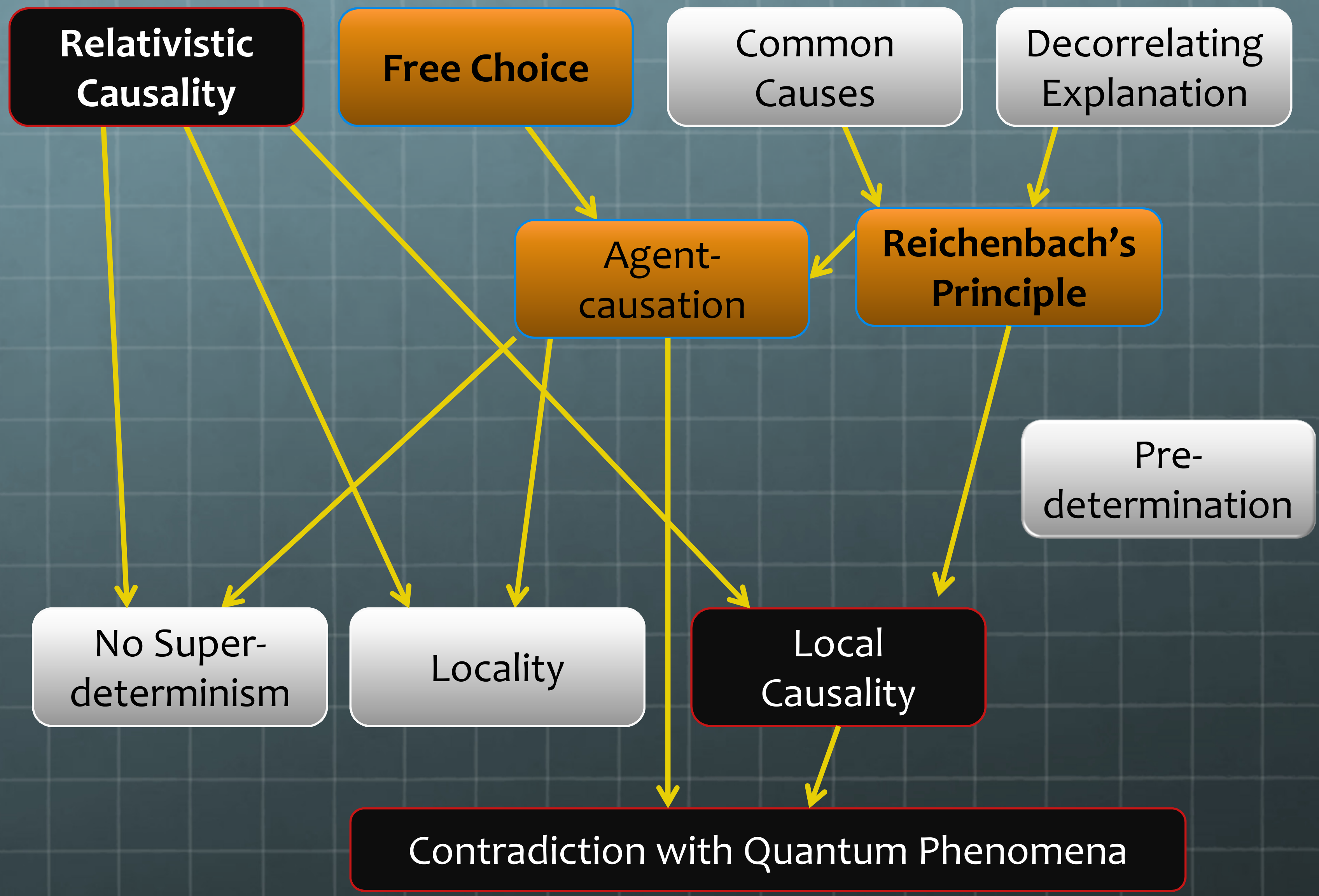
Reconciliation Version



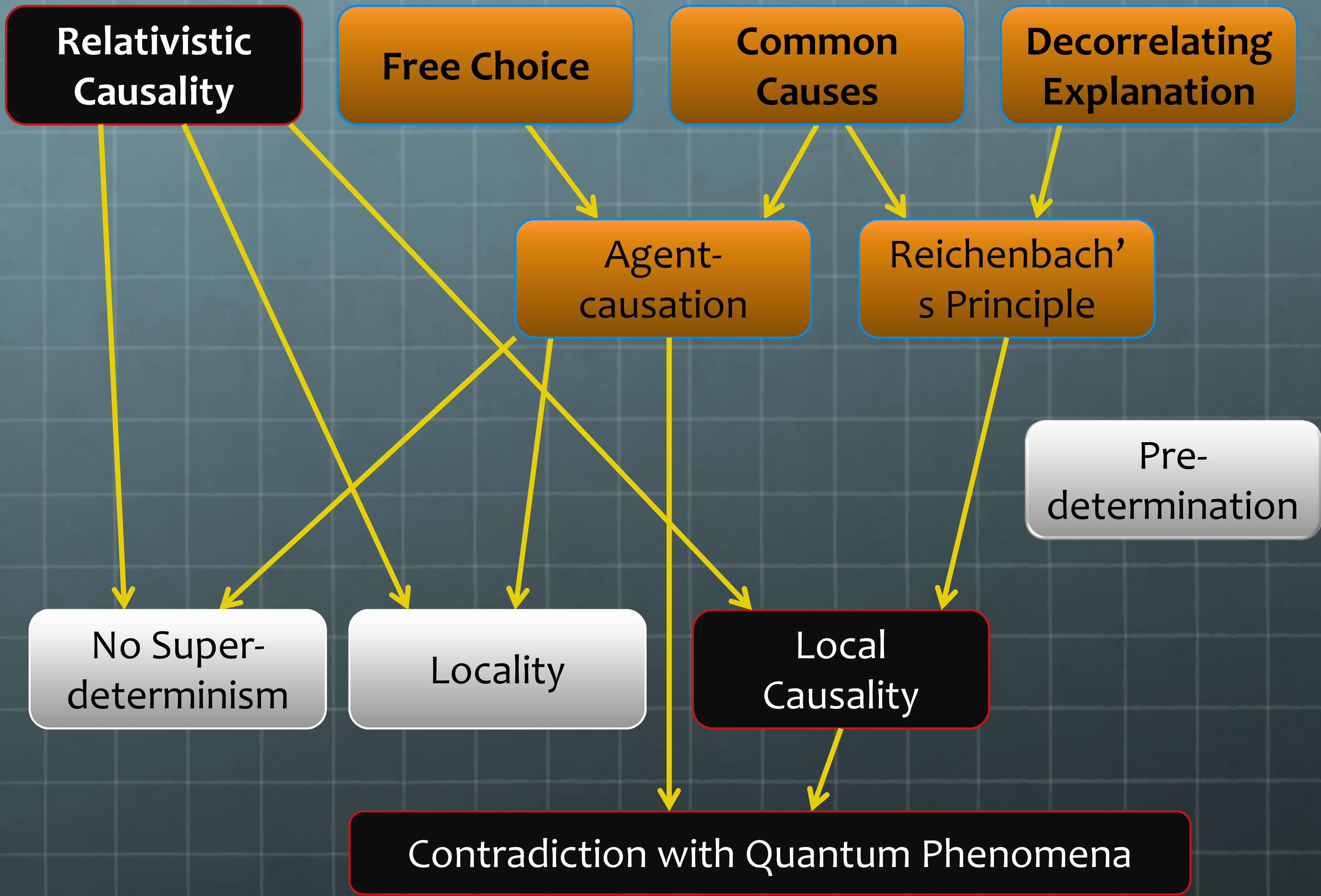
Original Realist Version (1976)



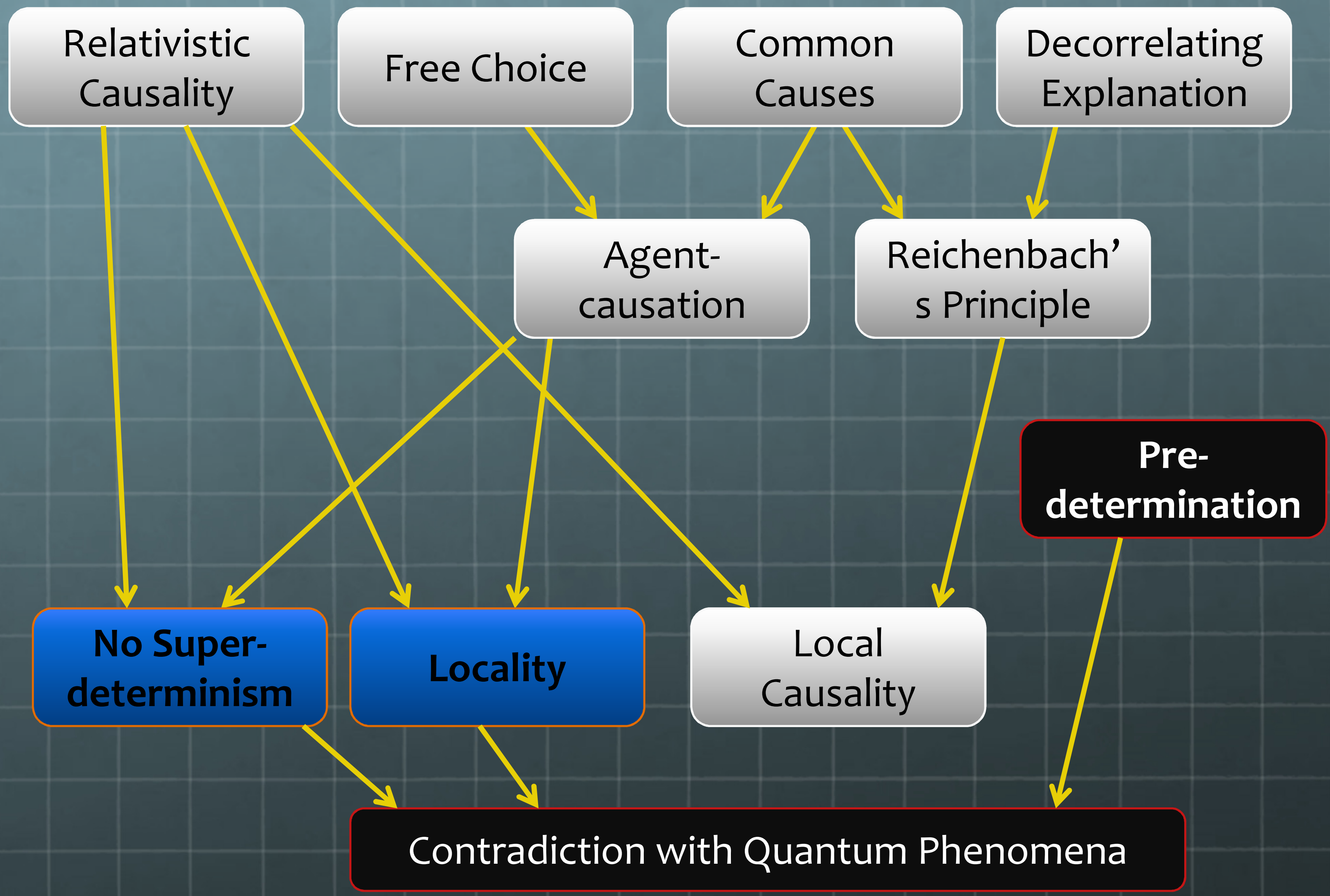
Intermediate Realist Version



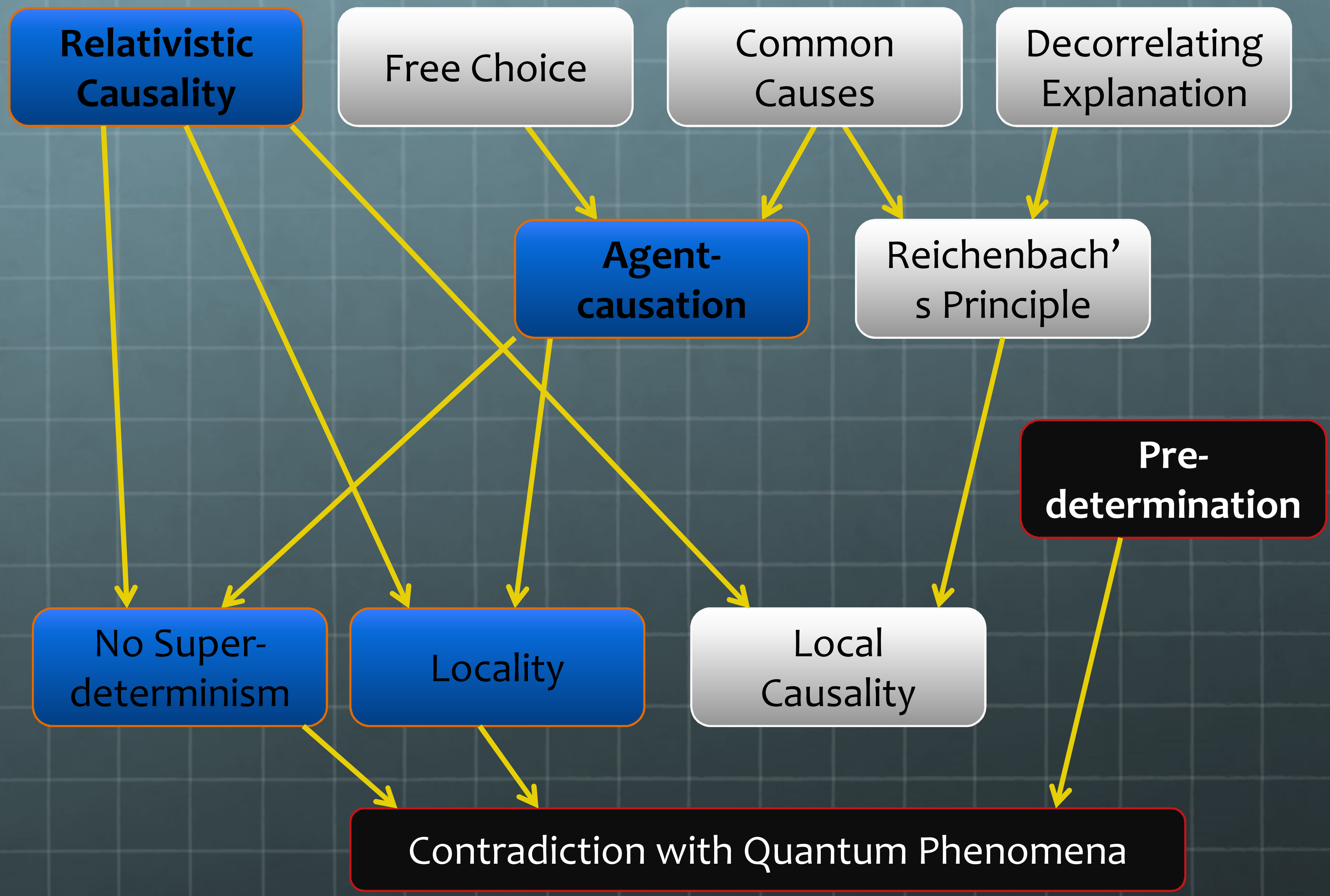
New Realist Version



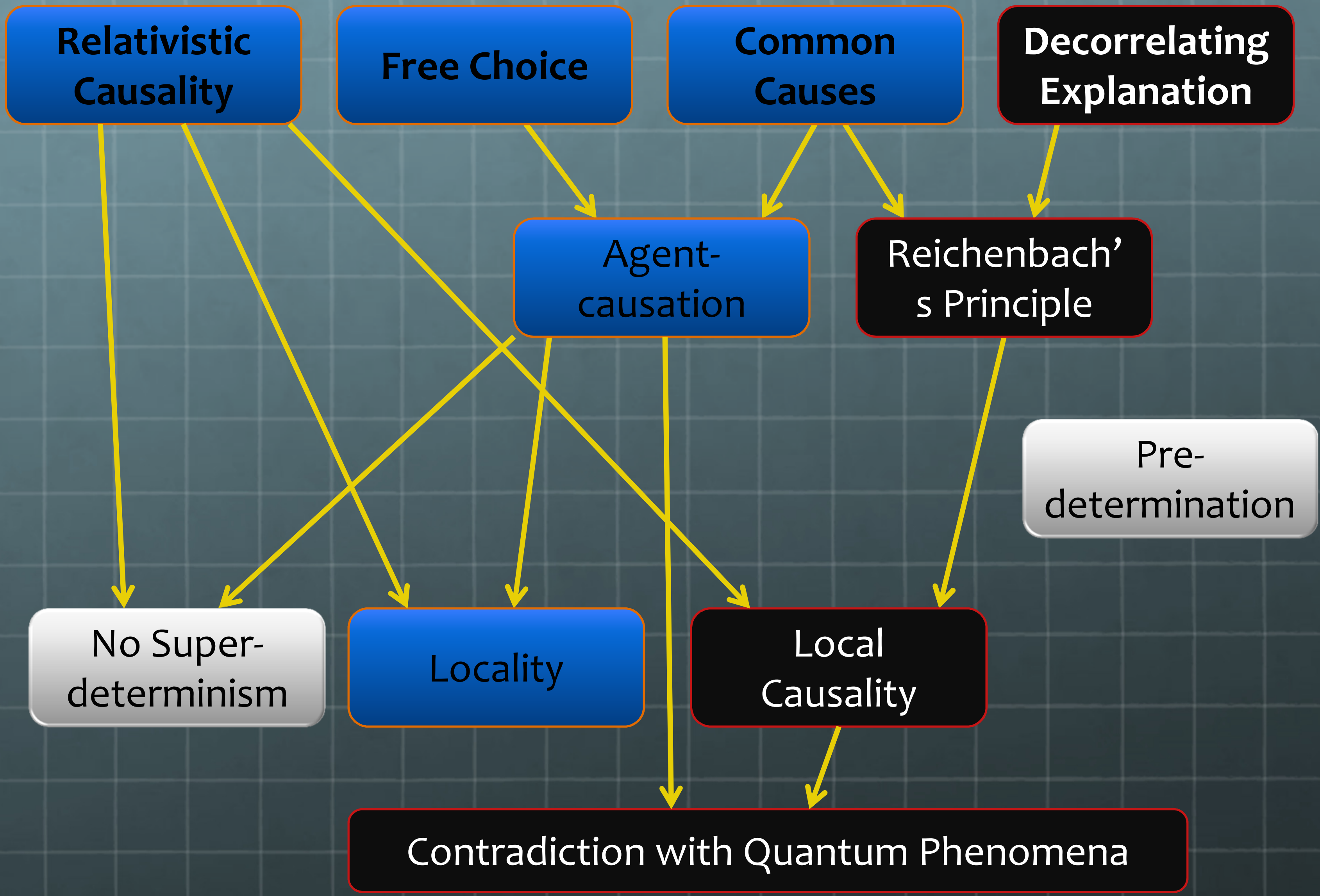
Original Operationalist Version (1964)



Intermediate Operationalist Version



New Operationalist Version



Considering causation confers …

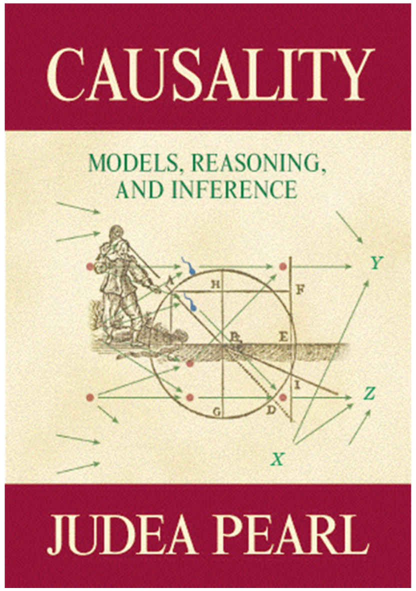
- A deeper understanding of why the two camps disagree, and why they prefer the different versions of Bell's theorem.
- A form of Bell's theorem that both camps might agree upon:
  - with terms not loaded with preconceptions;
  - in particular, without the problematic term "locality";
  - with an option for each camp of rejecting a principle (**Relativistic causality**, or **decorrelating explanation**).
- Several other conceptual advantages.

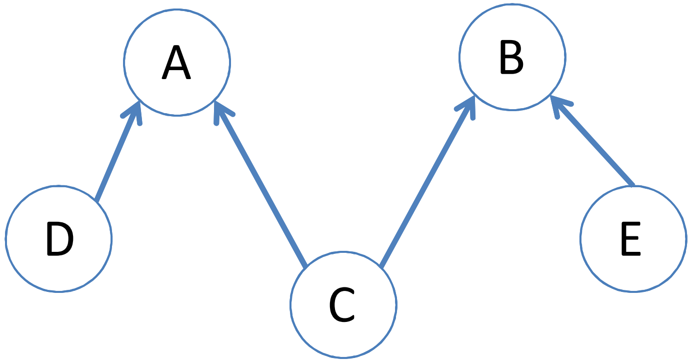
Taking stock

- There are actually two Bell's theorems (1964 and 1976), with essentially the same proof, but with different assumptions:
  - Bell 1964, favoured by operationalists
  - Bell 1976, favoured by realists
- By considering notions of causation we can:
  - Understand why the two camps disagree
  - Find a form of Bell's theorem both can agree upon.
- Still, we have to give up something important: macroreality; Minkowski space-time; the causal arrow of time; freedom of choice; relativistic causality; the common cause principle or the "decorrelating explanation" principle.
- → **The universe is profoundly strange …**

Going further

- Operationalist gains from going beyond Bell 1964:
  - Giving up Predetermination/Realism isn't enough
  - Modify Reichenbach's Principle → just giving it up isn't good enough!





Quantum causal models

- Joe Henson, Ray Lal and Matt Pusey, NJP **16**, 113043 (2014)
- Jacques Pienaar and Caslav Brukner, NJP **17**, 073020 (2015)

- Both provide generalisations of Reichenbach's principle that accommodate quantum correlations and reduce to the appropriate classical limit (in fact they do more than that!).

- Case closed for Bell's theorem?

There's still cause for concern

Reichenbach's principle is a special case of a more general principle, that we can call

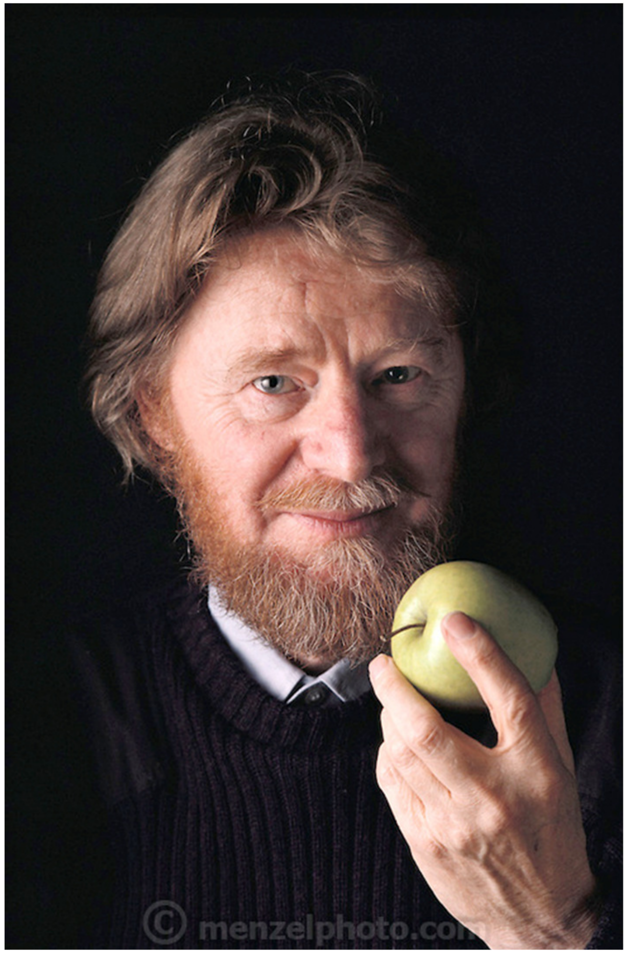
Causal Completeness:
"an event is *independent* of any events not among its effects given its causes".

Pienaar and Brukner propose a:

Quantum Causality Condition: "An *outcome* is independent of all *settings* that are not its causes and all *outcomes* that do not share a common cause".

Meanwhile, Henson *et al.* require a notion of "observed" and "unobserved" events.

“Settings”, “outcomes”: Who do we think “we” are?



“More importantly, the **“no signalling" notion rests on concepts which are desperately vague** . . . The assertion that “we cannot signal faster than light" immediately provokes the question: “Who do we think *we* are?” We who can make ‘measurements’, we who can manipulate ‘external fields’, we who can ‘signal’ at all, even if not faster than light? Do we include chemists, or only physicists, plants, or only animals, pocket calculators, or only mainframe computers?”

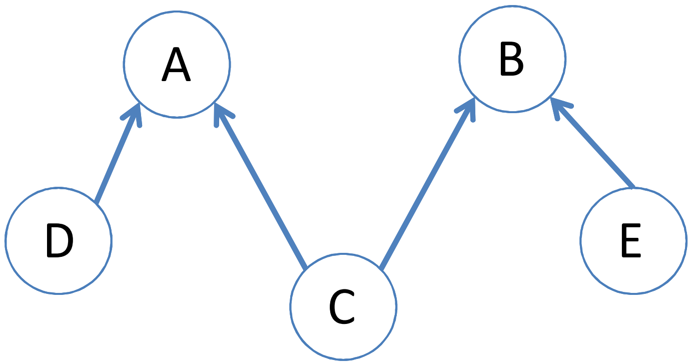
But who is "right"?

À la Quine: **Ontological relativity**

All consistent and empirically adequate descriptions of phenomena are justifiable models of reality.

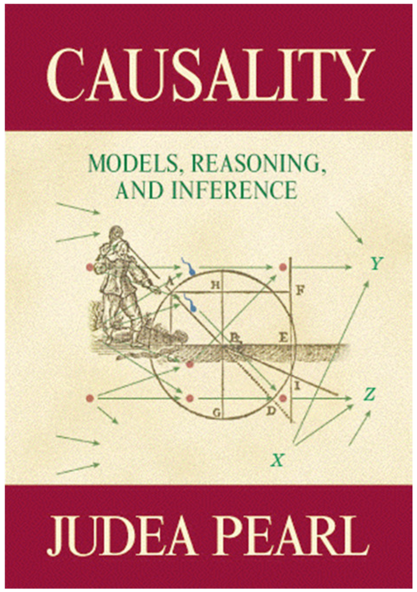
À la Wheeler: "In the end, the only law is that there is no law".

Directed acyclic graphs



"an event is *independent* of any events not among its effects given its causes"

E.g. $(B\perp AD|CE)$



Usually interpreted in terms of probabilistic independence:

$$(B\perp AD|CE) \Rightarrow P(B|ADCE) = P(B|CE)$$

➔ Reichenbach's principle → Bell's theorem

Semi-graphoid axioms

- Conditional independences should satisfy:
  - Symmetry $(X \perp Y | Z) \Leftrightarrow (Y \perp X | Z)$
  - Decomposition $(X \perp YW | Z) \Rightarrow (X \perp Y | Z)$
  - Weak Union $(X \perp YW | Z) \Rightarrow (X \perp Y | ZW)$
  - Contraction $(X \perp Y | Z) \ \& \ (X \perp W | ZY) \Rightarrow (X \perp YW | Z)$

Re-interpret "independence"

- Leifer and Spekkens: re-interpret independences in terms of conditional quantum states.

$$(B \perp A|C) \Rightarrow \rho_{B|AC} = \rho_{B|C}$$

- But, it's a work in progress. It gives "wrong" answers or is ill-defined in certain cases.

- **Conjecture**: There is no notion of 'independence' satisfying the semi-graphoid axioms that can restore causal completeness and relativistic causal structure for quantum phenomena.

  → "meta-Bell's theorem"

*M. S. Leifer and R. W. Spekkens, arXiv:1107.5849 [quant-ph]

Why do people care?

- “Bell’s theorem is the most profound discovery in science” – H. P. Stapp (1977).

- “Bell’s theorem is the most profound ramification of quantum theory that has been experimentally confirmed” – H. M. Wiseman (2014).

- “[Bell’s theorem] had a huge impact on our modern view of quantum theory [and is] at the heart of many protocols and applications in quantum information processing” – N. Brunner *et al.* (2014).

Summary of what implies what

Causal Principles:		Reichen-bach + Free Choice	➔	Agent Caus-ation	➔	?	➔	Manifest Agent Causation
	+ Minkowski Causality:	Local Causality	➔	Local Agency	➔	Locality	➔	Signal Locality
	[Classical physics]	✔		✔		✔		✔
~ EPR35	Orthodox QM	✖		✔ or ✖		✔		✔
Einstein 36	Psi-ontic QM	✖		✖		✔		✔
Bell 76	**Any** QT	✖		✔ or ✖		✔ or ✖		✔
??	Any QT with "realism" (IRS)	✖		✖		✔ or ✖		✔
~ Bell 64	Any QT with predetermined outcomes	✖		✖		✖		✔
Masanes et al. 2006	Post- QT with predictability	✖		✖		✖		✖

- Quine (1968) on the indeterminacy of translation:

“Here, gratuitously, we can systematically reconstrue our neighbor's apparent references to rabbits as really references to rabbit stages, and his apparent references to formulas as really references to Gödel numbers and vice versa. We can reconcile all this with our neighbor's verbal behavior, by cunningly readjusting our translations of his various connecting predicates so as to compensate for the switch of ontology”.

Principle of charity: “We will construe a neighbor's word heterophonically now and again if thereby we see our way to making his message less absurd”.

- Principle of Ontological Relativity
  - All consistent ontological descriptions of phenomena are correct descriptions of reality
  - Quine: confirmation holism, underdetermination of theory by data. “All consistent theories are equally justifiable”.
  - “**Here, gratuitously, we can systematically reconstrue our neighbor's apparent references to rabbits as really references to rabbit stages, and his apparent refer-ences to formulas as really references to Godel numbers and vice versa. We can reconcile all this with our neighbor's verbal behavior, by cunningly readjusting our translations of his various connecting predicates so as to compensate for the switch of ontology.** ”
  - “**Our usual domestic rule of translation is indeed the homophonic one, which simply carries each string of phonemes into itself; but still we are always prepared to temper homophony with what Neil Wilson has called the "prin-ciple of charity." 10 We will construe a neighbor's word heterophoni-cally now and again if thereby we see our way to making his message less absurd.** ”

- Principle of Ontological Relativity
  - All consistent ontological descriptions of phenomena are correct descriptions of reality
  - Quine: “Language is’

Reichenbach and Bell

Definition 1 (Principle of Common Cause (PCC)). *If two events A and B are correlated, i.e., if $P(A, B) > P(A)P(B)$, then either:*

- *(i) A and B are directly causally connected, i.e. either A causes B or B causes A, or*
- *(ii) A and B share a common cause that explains the correlation.*

Definition 2 (Factorisation of probabilities (FP)). *Two events have a common cause if and only if there exists a sufficient specification of variables λ corresponding to events in the common causal past of A and B such that conditioned on those variables the joint probability of A and B factorises:*

$$P(A, B|\lambda) = P(A|\lambda)P(B|\lambda) .$$

Reichenbach's Principle of Common Cause (R-PCC) = PCC + FP

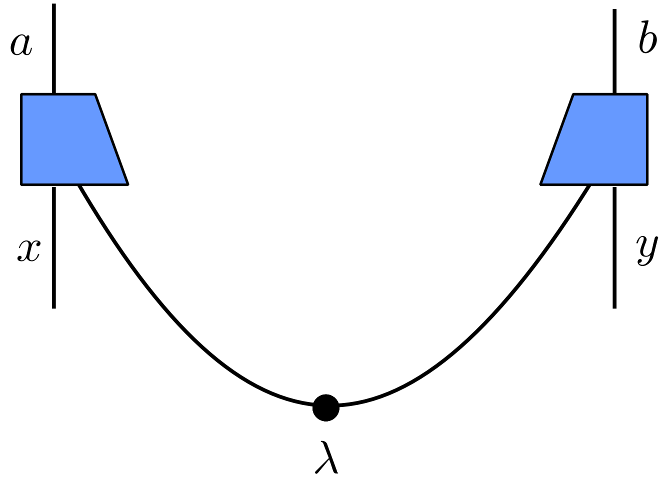
Reichenbach and Bell

Definition 3 (Relativistic causal structure (RCS)). *Events can be embedded in a single relativistic space-time. All causes of an event are to be found in the event's past light cone.*

Definition 4 (Law of Total Probability (LTP)). *Given a set of mutually exclusive events {x} whose probabilities sum to unity, the probability of an event y can be written as*

$$P(y) = \sum_x P(x)P(y|x)\,.$$

Bell scenario



Definitions 1-4 (and the assumption of free choice, $\mu(\lambda|x, y) = \mu(\lambda)$)
Imply a Local Hidden Variable (LHV) model:

$$P(a, b \,|\, x, y) = \sum_{\lambda} \mu(\lambda)\, P(a \,|\, x, \lambda) P(b \,|\, y, \lambda)$$

-> Bell inequalities -> inconsistency with quantum theory

Alternatives

- (a) Reject Principle of Common Cause
  - e.g. Van Fraassen [3], among others ("correlations require no explanation);
- (b) Reject Relativistic Causal Structure
  - e.g. Bohmian mechanics;
- (c) Reject Factorisation of Probabilities
  - e.g. Leifer and Spekkens
- (d) Reject Law of Total Probability
  - Hofer-Szabó and Vecsernyés "non-commutative common causes" (and L-S).

Approach	PCC	FP	RCS	LTP
H-V	✓	×	✓	×
L-S	✓	×	✓	✓
van Fraassen	×	×	✓	✓
Bohm	✓	✓	×	✓

Alternatives

- (a) Reject Principle of Common Cause
  - e.g. Van Fraassen [3], among others (“correlations require no explanation);
- (b) Reject Relativistic Causal Structure
  - e.g. Bohmian mechanics;
- (c) Reject Factorisation of Probabilities
  - e.g. Leifer and Spekkens
- (d) Reject Law of Total Probability
  - Hofer-Szabó and Vecsernyés “non-commutative common causes” (and L-S).

Cavalcanti and Lal, JPA **47**, 424018 (2014)

Approach	PCC	FP	RCS	LTP
~~H-V~~	~~✓~~	~~✓~~	~~✓~~	~~×~~
? L-S	✓	×	✓	✓
van Fraassen	×	×	✓	✓
Bohm	✓	✓	×	✓

"Non-commutative common causes"

Since Quantum Mechanics in general violates the LTP ("unperformed measurements have no results"), is sounds a priori plausible to reject it.

Hofer-Szabó and Vecsernyés propose such a modified form of Reichenbach's principle, in the language of Algebraic Quantum Field Theory.

Definition 5. A partition of the unit $\{C_k\}_{k\in K} \subset \mathcal{P}(\hat{\mathcal{A}})$ is said to be the *common cause system* of the commuting events $A, B \in \mathcal{P}(\hat{\mathcal{A}})$, which correlate in the state $\phi : \hat{\mathcal{A}} \to \mathbb{C}$, if for all $k \in K$ such that $\phi(C_k) \neq 0$, the following condition holds:

$$\frac{(\phi \circ E_c)(ABC_k)}{\phi(C_k)} = \frac{(\phi \circ E_c)(AC_k)}{\phi(C_k)} \frac{(\phi \circ E_c)(BC_k)}{\phi(C_k)} .$$

Where they define a "conditional expectation value" as

$$E_c(A) := \sum_{k\in K} C_k A C_k .$$

Unpacking non-commutative common causes

In the AQFT language, the probability of observing event A given an observation of C in state Á is

$$P_\phi(A|C) := \frac{\phi(CAC)}{\phi(C)}.$$

It is easy to see that

$$E_c(AC_{k'}) = C_{k'}AC_{k'}$$

The Eq. in Definition 5 then corresponds to

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)}$$

Which using Lüders' rule is

$$P_\phi(A, B|C_k) = P_\phi(A|C_k)P_\phi(B|C_k).$$

I.e., nothing but the screening-off condition in Def. 2.

Non-commutative common causes are trivial

- We can show that **any product state** can satisfy the requirements for a "non-commutative common cause.
- In fact they satisfy that for the correlations of ***any*** **quantum state**.
- Keeping FP while rejecting LTP leads to a **loss of deductive or causal link** between the "common causes" and their supposed effects. Non-commutative common causes do not *explain* the correlations.
- Satisfaction of definition 5 is best understood as meaning that *if* we were to measure the common cause, *then* the correlations would factorise; but that counterfactual does not in any way explain the actual non-factorisable correlations.