A new symmetry for quantum mechanics: trading drift and fluctuations in Entropic Dynamics

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Two attitudes about an emergent QM:

(1) Physical models represent Nature.

EmQM arises from coarse-graining a model that describes reality at a sub-quantum level.

(2) Physical models are a framework for processing information about Nature.

The dynamical laws are derived.

Focus directly on the relevant information that survives the coarse graining.

Our objective:

To derive Quantum Theory as Entropic Dynamics.

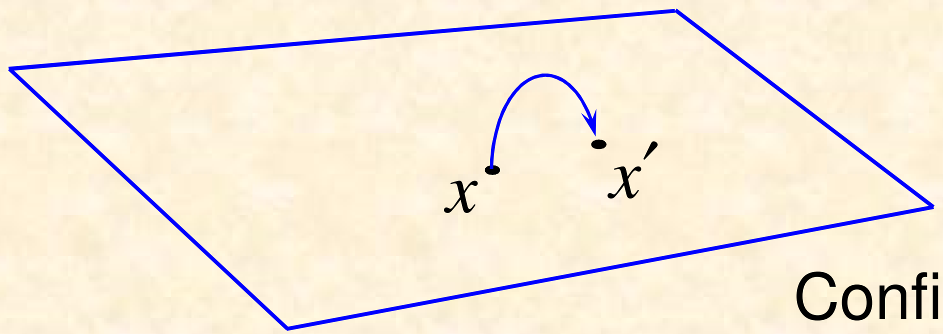
The subject matter:

The goal is to predict the positions of particles x.

(Or fields, or any other configurational variables).

Positions have definite but unknown values.

Dynamics:



First goal: find $P(x' \mid x)$

Entropic Dynamics

Maximize $$S[P,Q] = -\int dx'\, P(x'\,|\,x) \log \frac{P(x'\,|\,x)}{Q(x'\,|\,x)}$$

uniform

Constraints:

1) Motion is continuous. Impose short steps:

$$\langle \Delta x_n^a \Delta x_n^b \rangle \delta_{ab} = \kappa_n \qquad n = 1 \ldots N$$

2) Impose some directionality and correlation:

$$\langle \Delta x^A \rangle \partial_A \phi = \kappa' \qquad A = (n, a)$$

The result:

$$P(x' \mid x) = \frac{1}{\zeta} \exp[-\frac{1}{2}\sum_n \alpha_n \delta_{ab} \Delta x_n^a \Delta x_n^b + \alpha' \Delta x^A \partial_A \phi]$$

Displacement: $\Delta x^A = \langle \Delta x^A \rangle + \Delta w^A$

Expected drift : $\langle \Delta x_n^a \rangle = \frac{\alpha'}{\alpha_n} \delta^{ab} \frac{\partial \phi}{\partial x_n^b} \qquad \leftarrow O(\alpha_n^{-1})$

Fluctuations: $\langle \Delta w_n^a \rangle = 0$

$$\langle \Delta w_n^a \Delta w_n^b \rangle = \frac{1}{\alpha_n} \delta^{ab} \qquad \leftarrow O(\alpha_n^{-1/2})$$

Time:

The foundation of any notion of time is dynamics.

For large α_n the dynamics is all in the fluctuations:

$$\langle \Delta w_n^a \Delta w_n^b \rangle = \frac{1}{\alpha_n}\delta^{ab} = \frac{\eta}{m_n}\Delta t\,\delta^{ab}$$

clocks

mass

Define duration so that motion looks simple:

$$\alpha_n(x,t) = \frac{C_n}{\Delta t} \quad = \frac{m_n}{\eta\Delta t}$$

Entropic dynamics:

$$\rho(x',t') = \int dx\, P(x' \mid x)\, \rho(x,t)$$

Fokker-Planck equation: $\partial_t \rho = -\partial_A (\rho v^A)$

$$m_{AB} v^B = \partial_A \Phi$$

$$m_{AB} = m_n \delta_{ab} \qquad \Phi = \eta \alpha' \phi - \eta \log \rho^{1/2}(x,t)$$

Problem: this is standard diffusion, not QM!

Solution: allow the drift potential $\phi(x,t)$ to be dynamic.

E. Nelson (1979): non-dissipative diffusion

Define $\tilde{H}[\rho,\Phi]$ so that

$$\partial_t \rho = \frac{\delta \tilde{H}}{\delta \Phi} = -m^{AB}\partial_A\left(\rho\,\partial_B \Phi\right)$$

$$\tilde{H}[\rho,\Phi] = \int d^{3N}x\,\rho\,\frac{1}{2}m^{AB}\partial_A\Phi\partial_B\Phi + F[\rho]$$

Impose “energy” conservation

1) Hamilton’s equations

$$\partial_t \rho = \frac{\delta \tilde{H}}{\delta \Phi} \quad \text{and} \quad \partial_t \Phi = -\frac{\delta \tilde{H}}{\delta \rho}$$

2) Poisson brackets, symplectic structure,...

3) Action:

$$A = \int dt \left(\int \Phi \dot{\rho} - \tilde{H} \right)$$

Entropic Dynamics:

$$\partial_t \rho = \frac{\delta \tilde{H}}{\delta \Phi} = -m^{AB} \partial_A \left(\rho \, \partial_B \Phi \right) \qquad \text{FP}$$

$$-\partial_t \Phi = \frac{\delta \tilde{H}}{\delta \rho} = \frac{1}{2} m^{AB} \partial_A \Phi \partial_B \Phi + \frac{\delta F}{\delta \rho} \qquad \text{HJ}$$

??

Natural choice: $F[\rho] = \int dx \, \rho V \;\; + \xi \, m^{AB} I_{AB}$

$$\xi = \frac{\eta^2}{8}$$

$$I_{AB} = \int dx \frac{1}{\rho} \partial_A \rho \partial_B \rho$$

(Fisher)

Main result:

Combine ρ and Φ into $\Psi = \rho^{1/2} e^{i\Phi/\eta}$

to get QM:

$$i\eta\partial_t \Psi = -\frac{\eta^2}{2} m^{AB} \partial_A \partial_B \Psi + V\Psi$$

But.... this is an equation at the "macro" level.

What's going on at the "micro" level?

The constants α' and η have dropped out.

A symmetry:

A displacement: $\Delta x^A = \langle \Delta x^A \rangle + \Delta w^A$

Expected drift : $\langle \Delta x^A \rangle = \eta \alpha' m^{AB} \partial_B \phi \Delta t \quad = \eta' m^{AB} \partial_B \phi \Delta t$

Fluctuations: $\langle \Delta w^A \Delta w^B \rangle = \eta \, m^{AB} \Delta t \quad = \frac{\eta'}{\alpha'} m^{AB} \Delta t$

For large α' the fluctuations are suppressed and particles follow smooth trajectories along the probability flow lines.

→ the Bohmian limit

Conclusions:

- Entropic Dynamics provides a pathway to Hamiltonian dynamics and Action Principles.

- Quantum dynamics is a non-dissipative diffusion in configuration space.

- There is a family of models that differ at the “micro” level all leading to the same Schrödinger dynamics at the “macro” level.

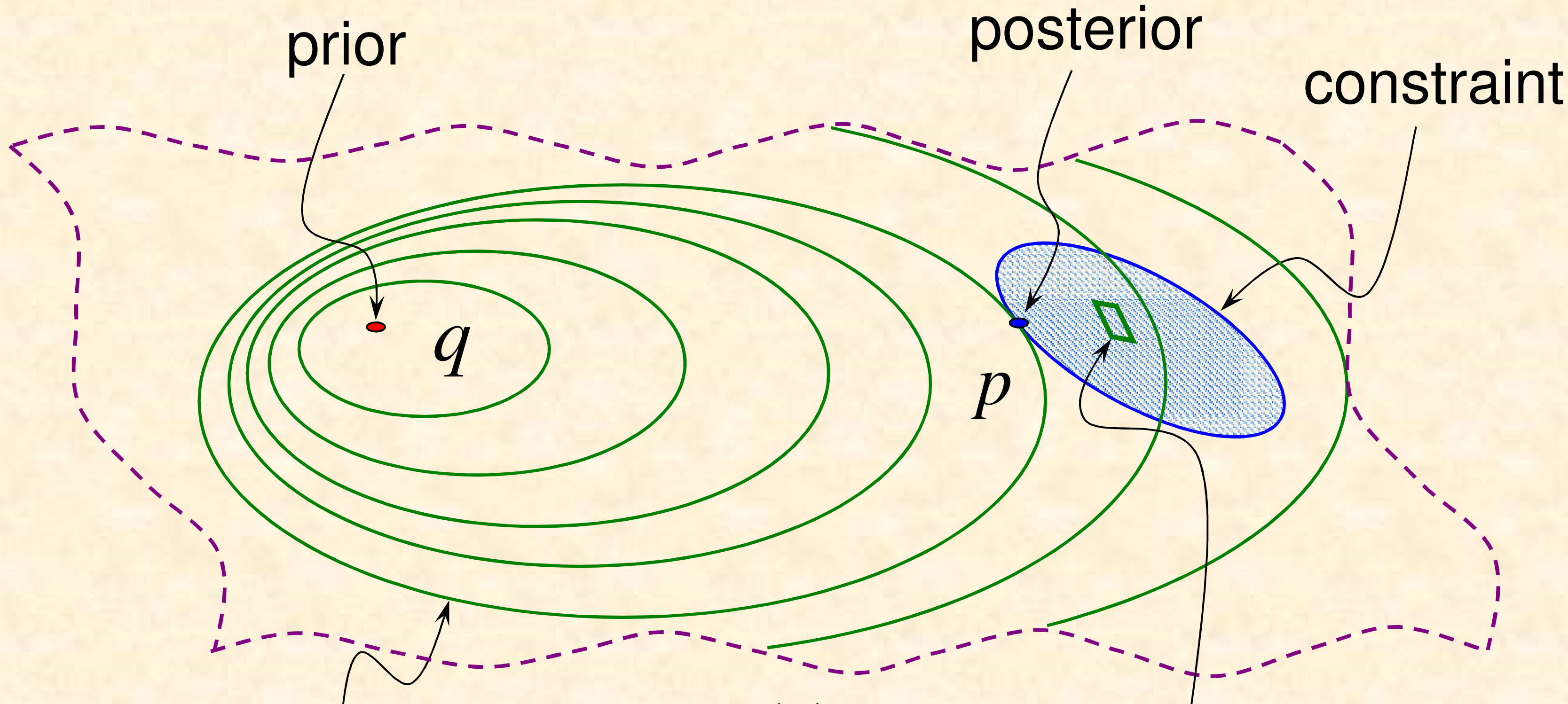
- Entropic Dynamics includes Bohmian Mechanics as a limiting case.

Thank you!

See Dan Bartolomeo's poster on

"Uncertainty Relations in the Entropic Dynamics approach to Quantum Mechanics"

Entropic Inference:



$$S[p,q] = -\int dx\, p(x) \log \frac{p(x)}{q(x)} \qquad \Pr(dV) \propto e^{S[p,q]} dV$$

Maximize $S[p,q]$ subject to the appropriate constraints.

(MaxEnt, Bayes' rule and Large Deviations are special cases.)

Impose "energy" conservation

$$\frac{d\tilde{H}}{dt} = \int dx \left[\frac{\delta\tilde{H}}{\delta\Phi}\partial_t\Phi + \frac{\delta\tilde{H}}{\delta\rho}\partial_t\rho \right] = 0$$

(with $\frac{\delta\tilde{H}}{\delta\Phi} = \partial_t\rho$)

$$\frac{d\tilde{H}}{dt} = \int dx \left[\partial_t\Phi + \frac{\delta\tilde{H}}{\delta\rho} \right] \partial_t\rho = 0$$

$$\Rightarrow \quad \partial_t\rho = \frac{\delta\tilde{H}}{\delta\Phi} \quad \text{and} \quad \partial_t\Phi = -\frac{\delta\tilde{H}}{\delta\rho}$$

Three ingredients:

E. Jaynes — entropic inference

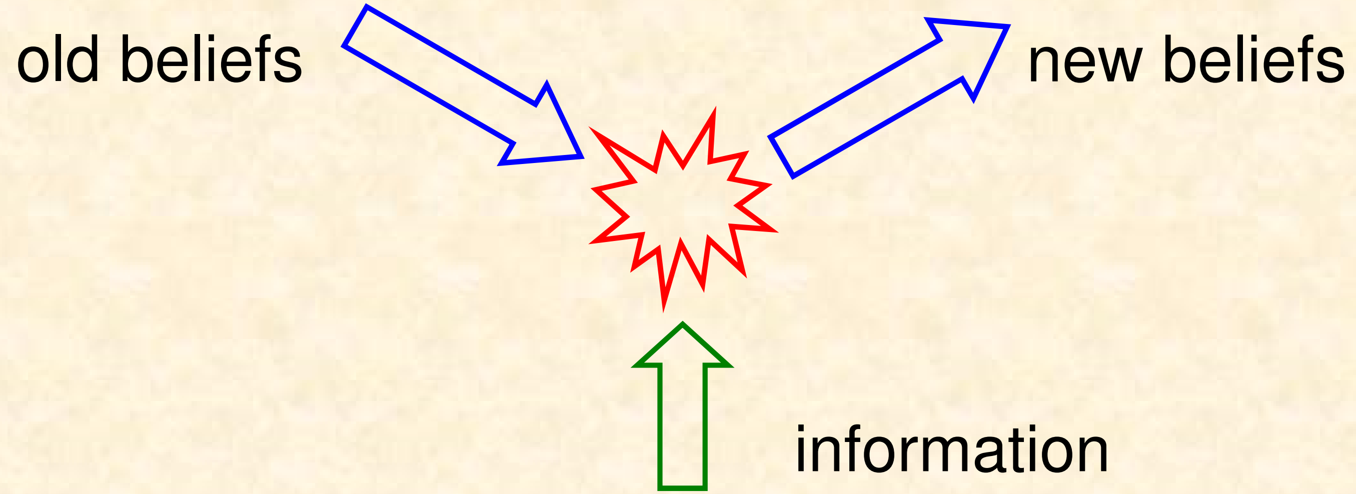
E. Nelson — non-dissipative diffusion

J. Barbour — relational time

⇓

entropic dynamics

Inference is dynamics too!



Information is what induces the change in rational beliefs.

Information is what constrains rational beliefs.