TWO-ELECTRON SYSTEM CORRELATED BY THE ZERO-POINT FIELD

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Which is the physical agent behind the antisymmetry of the electron state vectors?

- SED: linear resonant response of the electron to certain modes of the random ZPF
- Two-particle system: resonant response to common field modes
- The field as entangler
- Correlation between the entangling field modes

A quick run-through:

SED in the quantum regime

Equation of motion:

$$m\ddot{x} = f(x) + m\tau\dddot{x} + eE(t)$$

with

$$E(t) = \sum_k E_k a_k e^{i\omega_k t}$$, random amplitudes $a(\omega_k) = \exp i\zeta(\omega_k)$.

Average energy per field mode $\mathcal{E}(\omega_k) = \hbar\omega_k/2$.

Stationary state α: linear resonant response to a set of field modes,

$$\omega_{\alpha\beta} = (\mathcal{E}_\alpha - \mathcal{E}_\beta)/\hbar, \quad a_{\alpha\beta} = e^{i\zeta_{\alpha\beta}},$$

$$\boxed{F_\alpha(t) = \sum_\beta F_{\alpha\beta} a_{\alpha\beta} e^{i\omega_{\alpha\beta} t}.}$$

Under conditions of ergodicity:

Chain rule $a_{\alpha\beta}a_{\beta\gamma} = a_{\alpha\gamma}$, i.e., $a_{\alpha\beta} = e^{i\zeta_{\alpha\beta}} = e^{i(\varphi_\alpha - \varphi_\beta)}$.

Ensemble average equal to the *quantum average*:

$$\overline{F_\alpha} = \sum_\beta F_{\alpha\beta} a_{\alpha\beta} \delta_{\alpha\beta} = \langle\alpha| \widehat{F} |\alpha\rangle \, .$$

For **two particles** subject to the *same* background field

$$(FG)_A(t) = \sum_B (FG)_{AB} b_{AB} e^{i\omega_{AB} t},$$

where $A = (\alpha, \alpha')$, composite state, and

particle 1

$$(FG)_{AB} = F_{\alpha\beta} G_{\alpha'\beta'} \, .$$

particle 2

Two-particle states

The composite coefficients

$$b_{AB} = a_{1\alpha\beta} a_{2\alpha'\beta'}$$

automatically satisfy the chain rule $b_{AC} = b_{AB} b_{BC}$,

and $\omega_{AB} = \omega_{\alpha\beta} + \omega_{\alpha'\beta'}$.

Under conditions of ergodicity,

$$\overline{FG}_A = \sum_B (FG)_{AB} \lambda_{AB}$$

where

$$\lambda_{AB} = b_{AB}|_{\omega_{AB}=0} \, .$$

Notice that $\omega_{AB} = 0$ does not necessarily mean *A* = *B*.

When states *A* and *B* are degenerate

$$\mathcal{E}_A = \mathcal{E}_B \quad \text{but} \quad \mathcal{E}_\alpha \neq \mathcal{E}_\beta,$$

whence

$$\overline{FG}_A = (FG)_{AA} + \sum_{B \neq A}^{\mathcal{E}_B = \mathcal{E}_A} (FG)_{AB} \lambda_{AB}.$$

For two identical particles there is always degeneracy:

$$\overline{FG}_A = (FG)_{AA} + (FG)_{AB} \lambda_{AB},$$

$$\overline{FG}_B = (FG)_{BB} + (FG)_{BA} \lambda_{BA},$$

with

$$\lambda_{AB} = \lambda_{BA}^* = b_{AB}|_{\omega_{AB}=0}$$

The vacuum field as entangler

In standard notation,

$$\overline{FG}_A = \langle A | \widehat{FG} | \psi_{AB} \rangle \,,$$

$$\overline{FG}_B = \lambda_{BA} \langle B | \widehat{FG} | \psi_{AB} \rangle \,,$$

where

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}} \Big(|A\rangle + \lambda_{AB} |B\rangle \Big)$$

so that

$$\overline{(FG)}_{AB} = \frac{1}{2} \left[\overline{FG}_A + \overline{FG}_B \right] = \langle \psi_{AB} | \widehat{FG} | \psi_{AB} \rangle$$

gives the statistical average $\overline{FG}$. Notice that λ_{AB} appears in the covariance only if *F* and *G* have nondiagonal elements:

$$\Gamma_{FG} = \frac{1}{4}(F_{\alpha\alpha} - F_{\beta\beta})(G_{\alpha'\alpha'} - G_{\beta'\beta'}) + \lambda_{AB} \mathrm{Re} F_{\alpha\beta} G_{\alpha'\beta'}$$

Correlation between the entangling field modes

An entangled state with $S_z = 0$ has the general form

$$|\psi\rangle^S_{nm0} = |\phi\rangle_{nm}\, |\chi\rangle^S_0$$

with

$$|\phi\rangle_{nm} = \tfrac{1}{\sqrt{2}}(|n\rangle\, |m\rangle + \lambda^o\, |m\rangle\, |n\rangle),$$

$$|\chi\rangle^S_{s_z=0} \equiv |\chi\rangle^S_0 = \tfrac{1}{\sqrt{2}}(|+\rangle\, |-\rangle + \lambda^s\, |-\rangle\, |+\rangle).$$

Antisymmetry requires that $\lambda^o \lambda^s = -1$, which means

$$\lambda_{AB} = a_{1m-n+} a_{2n+m-} = -1$$

with

$$|A\rangle = |n\rangle\, |m\rangle\, |+\rangle\, |-\rangle\,, \quad |B\rangle = |m\rangle\, |n\rangle\, |-\rangle\, |+\rangle$$

This holds also for $S_z = \pm 1$.

Hence in general,

$$\lambda_{AB} = a_{1\alpha\beta} a_{2\beta\alpha} = -1,$$

where α, β refer to the whole set of (one-particle) quantum numbers.

We thus conclude that the active zero-point field modes that sustain an entangled state are always anticorrelated.

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General reference:

The Emerging Quantum. The Physics Behing Quantum Mechanics. L. de la Peña, A. M. Cetto and Andrea Valdés-Hernández, Springer Verlag, Berlin, 2015.

Thanks for your attention