Models of spontaneous wave function collapse: what they are, and how they can be tested.
Collapse models: What they are

They are nonlinear and stochastic (phenomenological) modifications of the Schrödinger equation, which include the collapse of the wave function

\[
d|\psi\rangle_t = \left[ -\frac{i}{\hbar} H dt + \sqrt{\lambda} (A - \langle A \rangle_t) dW_t - \frac{\lambda}{2} (A - \langle A \rangle_t)^2 dt \right] |\psi\rangle_t
\]

quantum collapse

\[
\langle A \rangle_t = \langle \psi_t | A | \psi_t \rangle \rightarrow \text{nonlinear}
\]

The wave function is dynamically and stochastically driven by the noise \(W_t\) towards one of the eigenstates of the operator \(A\).

This form is fixed by the requirement of no-faster-than-light signaling and norm conservation (Adler’s book on Trace Dynamics).
No one really knows.
But the gravitational coupling to matter is the right one for the collapse...

almost the right one

\[
\frac{d}{dt} |\psi_t\rangle = \left[ -\frac{i}{\hbar} H + \int d^3x \, \hat{M}(x) h(x, t) + O(\hat{M}, \hbar) \right] |\psi_t\rangle
\]

**Anti-Hermitian** coupling between mass density and gravity \(\rightarrow\) no grav.

waves  
(S.L. Adler, arXiv:1401.0353)

A different theory of gravity would be needed.

**Side note on the Schrödinger-Newton equation:** it is not a collapse equation in the sense of collapse models. But it does collapse the wave function, and it could serve to discriminate whether gravity is quantum or fundamentally classical

(A. Großardt et al., ArXiv:1510.01696)
## CSL model and its variations

### White noise models
- All frequencies appear with the same weight

#### GRW / CSL

#### QMUPL
- DP

### Colored noise models
- The noise can have an arbitrary spectrum

#### Non-Markovian CSL

#### Non-Markovian QMUPL

### Infinite temperature models
- No dissipative effects

### Finite temperature models
- Dissipation and thermalization

#### Dissipative QMUPL

#### Dissipative GRW & CSL

#### Non-Markovian & dissipative QMUPL
(Mass-proportional) CSL model


\[
\frac{d}{dt} |\psi_t\rangle = \left[ -\frac{i}{\hbar} H + \frac{\sqrt{\gamma}}{m_0} \int d^3 x (M(x) - \langle M(x) \rangle_t) dW_t(x) \\
- \frac{\gamma}{2m_0^2} \int \int d^3 x d^3 y G(x - y) (M(x) - \langle M(x) \rangle_t) (M(y) - \langle M(y) \rangle_t) \right] |\psi_t\rangle
\]

\[M(x) = ma^\dagger(x)a(x)\quad G(x) = \frac{1}{(4\pi r_C)^3/2} \exp[-(x)^2/4r_C^2]\]

\[w_t(x) \equiv \frac{d}{dt} W_t(x) = \text{noise}\quad \mathbb{E}[w_t(x)] = 0\quad \mathbb{E}[w_t(x)w_s(y)] = \delta(t - s)G(x - y)\]

Two parameters

\[\gamma = \text{collapse strength}\quad r_C = \text{localization resolution}\]

\[\lambda = \gamma/(4\pi r_C^2)^{3/2} = \text{collapse rate}\]
The collapse rate

Small superpositions

\[ \bullet + \bullet \]

\[ \ll r_C \]

Collapse NOT effective

Large superpositions

\[ \bullet + \bullet \]

\[ \geq r_C \]

Collapse effective

\[ \Gamma = \lambda n^2 N \]

Amplification mechanics

Few particles
-
no collapse quantum behavior

Many particles
-
Fast collapse classical behavior

\( n = \text{number of particles within } r_C \)

\( N = \text{number of such clusters} \)
The collapse rate of the CSL model

**Microscopic world**
(few particles)

\[ \lambda \sim 10^{-8\pm2} \text{s}^{-1} \]

**Mesoscopic world**
Latent image formation
+ perception in the eye
(\( \sim 10^4 - 10^5 \) particles)

S.L. Adler, JPA 40, 2935 (2007)
A. Bassi, D.A. Deckert & L. Ferialdi, EPL 92, 50006 (2010)

**Macroscopic world**
(> \( 10^{13} \) particles)


\[ r_C = \frac{1}{\sqrt{\alpha}} \sim 10^{-5} \text{cm} \]
Matter-wave interferometry
world mass record: $10^4$ amu (Vienna, 2013)

Brief history of matter-wave interferometry:

- **C$_{60}$ (720 AMU)**

- **C$_{70}$ (840 AMU)**

- **C$_{30}$H$_{12}$F$_{30}$N$_2$O$_4$ (1,030 AMU)**

- **Largest Molecule (10,000 AMU)**
  S. Eibenberger *et al*., PCCP 15, 14696 (2013)

**NANOQUESTFIT: $10^5$ AMU**
EU Project under FP7

**Future experiments: $\sim 10^6$ AMU**
K. Hornberger *et al*., Rev. Mod. Phys. 84, 157 (2012)
P. Haslinger *et al*., Nature Phys. 9, 144 (2013)

**MAQRO** consortium for a space mission with ESA (micro-gravity)
Matter-wave interferometry
Upper bounds on the collapse parameters

The exclusion zone is pretty much insensitive to the type of collapse model: CSL, dissipative CSL, non-Markovian CSL.

Matter-wave interferometry provides a general test of collapse models

M. Toros and A. Bassi, work in progress
One needs to introduce mass proportionality in the model
Adler’s value for $\lambda$ is ruled out by 3 orders of magnitude, unless the noise spectrum has a cut off below $10^{18}$ Hz. (ArXiv 1501.04462)

Strongest upper bound on the collapse parameter $\lambda$
Non interferometric tests with opto-mechanical systems


Qualitative behavior

Quantitative behavior
Experimental bounds from non-interferometric tests

**Cantilever:**
OK for CSL and non-Markovian CSL. Probably not OK for dissipative CSL, with $T = 1K$. ArXiv:1510.05791

**X-ray:**
OK for CSL and dissipative CSL. Not OK for non-Markovian CSL, with cutoff below $10^{18} \text{ Hz}$ ArXiv:1501.04462

A. Vinante et al., ArXiv 1510.05791
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