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back to namelist

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Regularization of derivatives and fractional approximation of non-differentiable trajectories

A central notion of physics is the rate of change. This perception inspired Newton and Leibniz to develop the apparatus of differential calculus. This calculus is not limited only to linear rates of change, e.g. to the concept of derivatives as mathematical idealizations of the linear growth. Fractional calculus has been also developed with the idea to describe the rate of change of strongly non-linear phenomena, such as the phenomena governed by power laws. Yet in another recent development of fractional calculus the link with localizable approaches has been explored. Classical physics variables, such as velocity or acceleration, are considered to be differentiable functions of position. On the other hand, quantum mechanical paths were found to be non-differentiable and stochastic. The relaxation of the differentiability assumption opens new avenues in describing physical phenomena, for example, using the scale relativity theory developed by Nottale, which assumes strong non-linearity and factuality of quantum-mechanical trajectories. The main application of the presented approach comprises a formal regularization procedure for the derivatives of Holderian functions, which allows for removal of the weak singularity in the derivative caused by strong non-linearities. Moreover using the same approach, generalized velocities (i.e. alpha-velocities) can be also defined on fractal curves. Some theoretical results related to singular fractal curves will be presented. Possible applications of presented approach are regularizations of quantum mechanical paths and Brownian motion trajectories, which are Holder ½.

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